CS 277: Control and Reinforcement Learning

Winter 2024

Lecture 6: Advanced Model-Free RL

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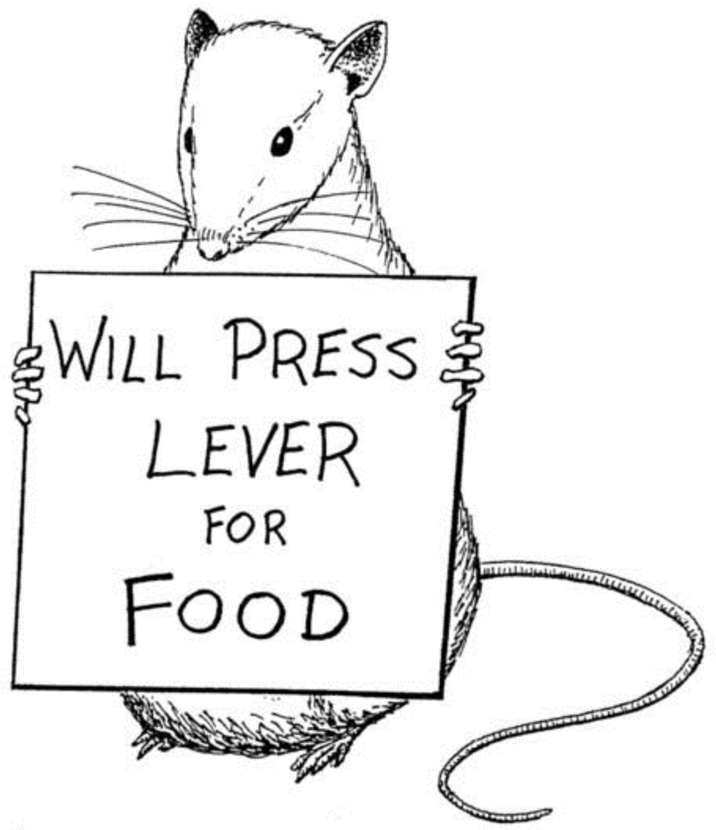


Logistics

assignments

- Quiz 3 due next Monday
- Exercise 2 due the following Monday

Today's lecture

Advantage estimation

Convergence of RL

Continuous action spaces

Trust-region methods

Baselines

- Constant shift b in return doesn't matter for the policy gradient

$$\mathbb{E}_{\xi\sim p_\theta}[(R(\xi)-b)\nabla_\theta \log p_\theta(\xi)] = \nabla_\theta \mathbb{E}_{\xi\sim p_\theta}[R(\xi)-b] = \nabla_\theta J_\theta$$

$\mathbb{E}[b]=b$ **independent of** θ

- But it can make a huge difference in its variance
  - Consider $\mathbb{E}[xy]$ vs. $\mathbb{E}[x(y+100)]$, with uniform $(x,y)\in\{-1,1\}^2$
- Making $y-b$ zero-mean (minimum $\mathbb{V}[y-b]$) is a good rule of thumb:
  - Update $b \to R(\xi)$ (approaches the expected return)
  - Estimate $\nabla_\theta J_\theta \approx (R(\xi)-b)\nabla_\theta \log p_\theta(\xi)$

State-dependent baselines

- What can b depend on?

$$\mathbb{E}_{(s_t,a_t)\sim p_\theta}[b\,\nabla_\theta \log \pi_\theta(a_t \mid s_t)] = \mathbb{E}_{s_t\sim p_\theta}[\,\nabla_\theta \mathbb{E}_{(a_t|s_t)\sim\pi_\theta}[b]] = 0$$

  - As long as b is independent of a_t given s_t (i.e. not caused by $\pi_\theta(a_t \mid s_t)$)

- Updating $b(s_t) \to R_{\geq t}(\xi) \Rightarrow$ we're learning $V_{\pi_\theta} = \mathbb{E}[R_{\geq t}(\xi) \mid s_t]$

- In the TD PG version:

$$\nabla_\theta J_\theta = \sum_t \gamma^t \mathbb{E}_{(s_t,a_t)\sim p_\theta}[(\underbrace{Q_{\pi_\theta}(s_t, a_t)}_{\text{future return estimator for action } a_t} - \underbrace{V_{\pi_\theta}(s_t)}_{\text{baseline}})\,\nabla_\theta \log \pi_\theta(a_t \mid s_t)]$$

Advantage estimation

- Advantage function: $A_\pi(s,a) = Q_\pi(s,a) - V_\pi(s)$
- AC PG with baseline: $\nabla_\theta J_\theta = \sum_t \gamma^t \mathbb{E}_{(s_t,a_t)\sim p_\theta}[A_{\pi_\theta}(s_t,a_t)\nabla_\theta \log \pi_\theta(a_t|s_t)]$
- How to estimate $A(s,a)$ using a critic $V_\phi(s)$?
  - MC:

$$A(s_t,a_t) \approx R_{\geq t}(\xi) - V_\phi(s)$$

  - TD:

$$A(s,a) \approx r + \gamma V_\phi(s') - V_\phi(s)$$

Advantage Actor–Critic (A2C)

MF

θ

DP

π'

max

Algorithm Advantage Actor–Critic

Initialize π_θ and V_ϕ

repeat

Roll out $\xi \sim p_\theta$

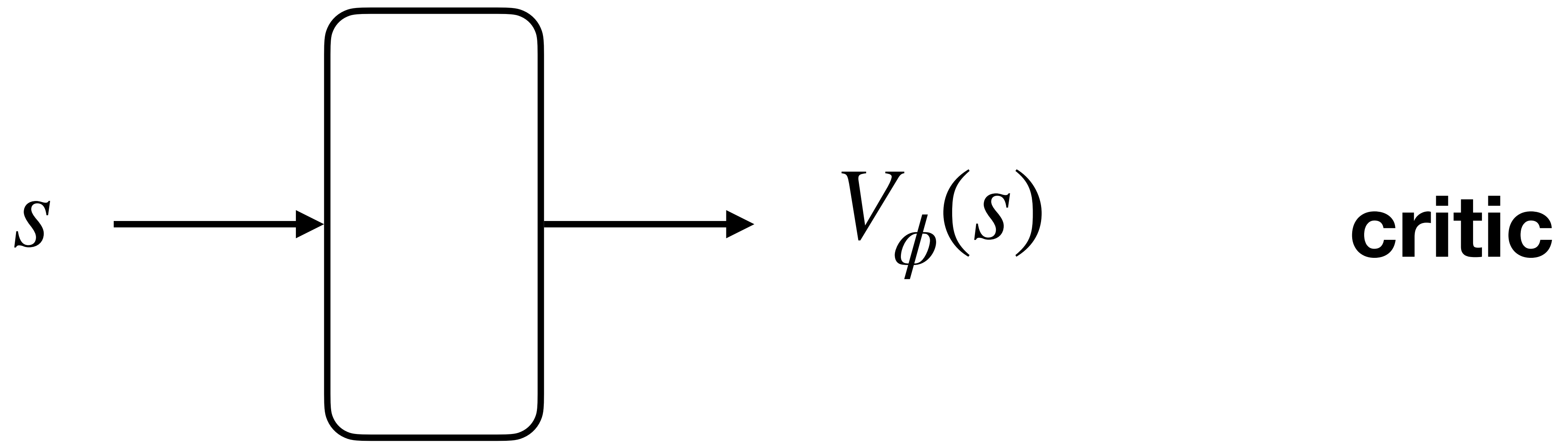
Update $\Delta\theta \leftarrow \sum_t (R_{\geq t}(\xi) - V_\phi(s_t))\nabla_\theta \log \pi_\theta(a_t|s_t)$

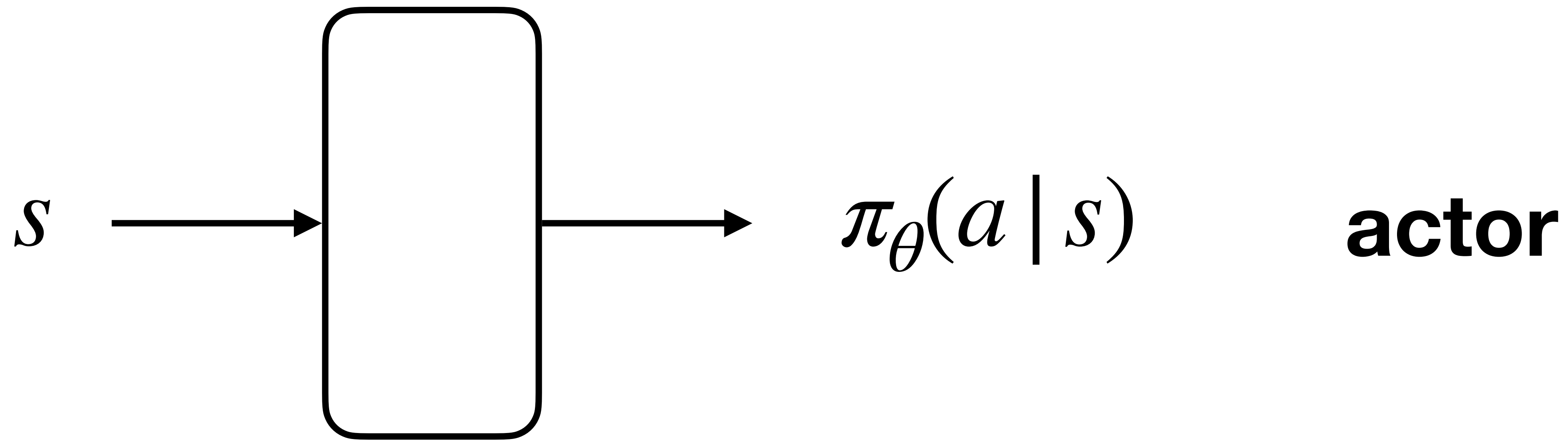
Descend $L_\phi = \sum_t (R_{\geq t}(\xi) - V_\phi(s_t))^2$

[Mnih et al., 2016]

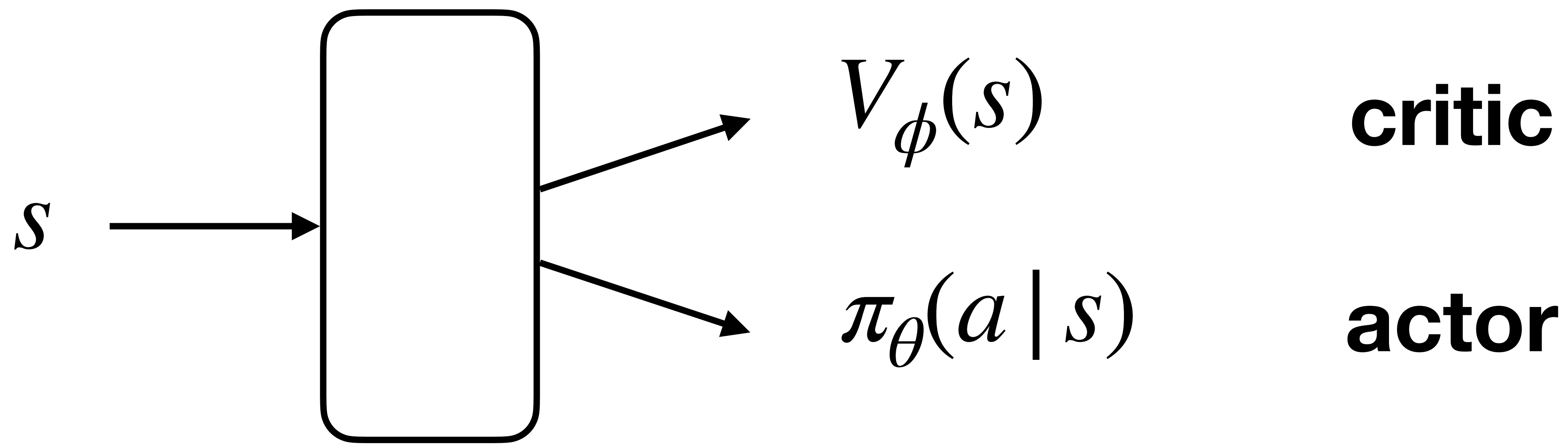
Practical considerations: param sharing

- Separate parameters:



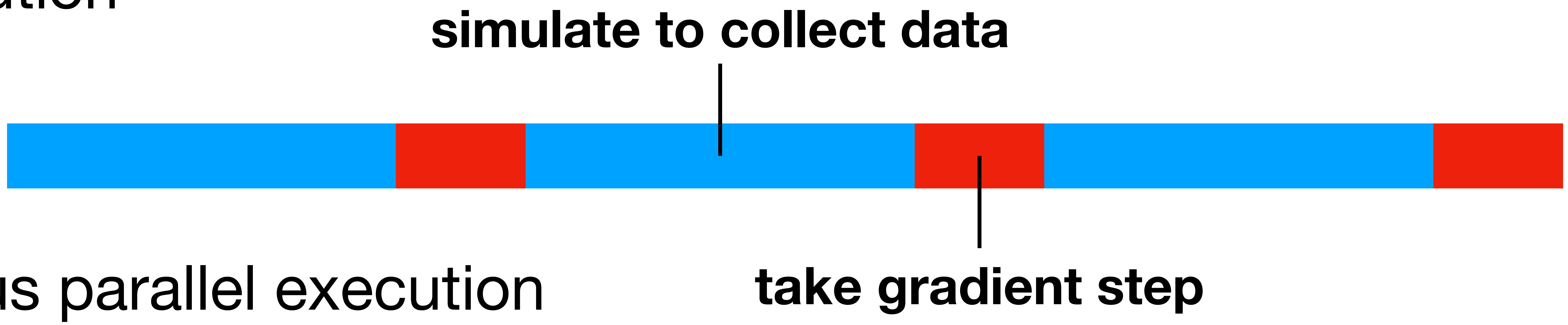


- Shared parameters:
  - Can be more data efficient
  - Can be less stable

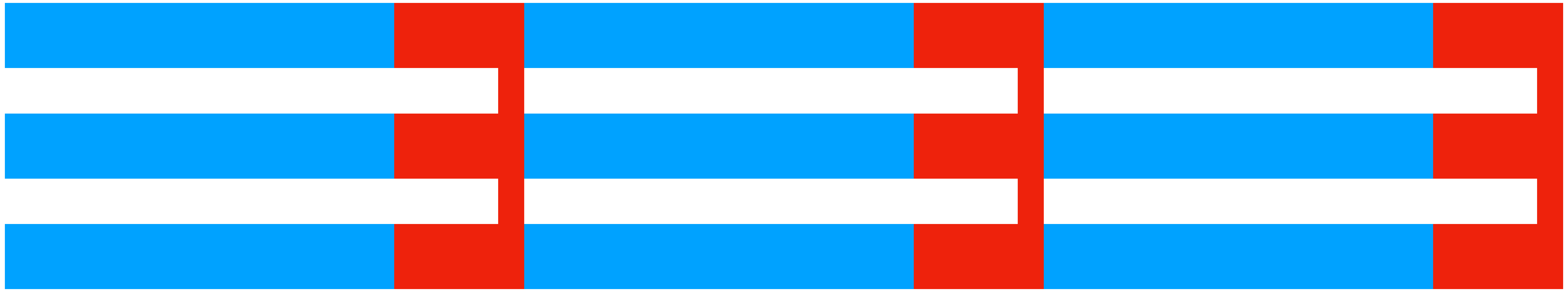


Practical considerations: distributed comp.

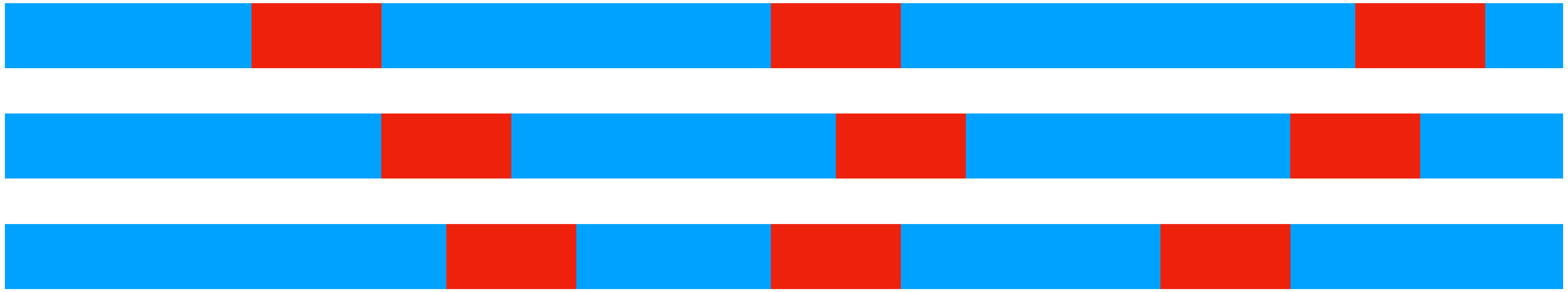
- Serial execution



- Synchronous parallel execution



- Asynchronous parallel execution (A3C)



[Mnih et al., 2016]

Comparing advantage estimators

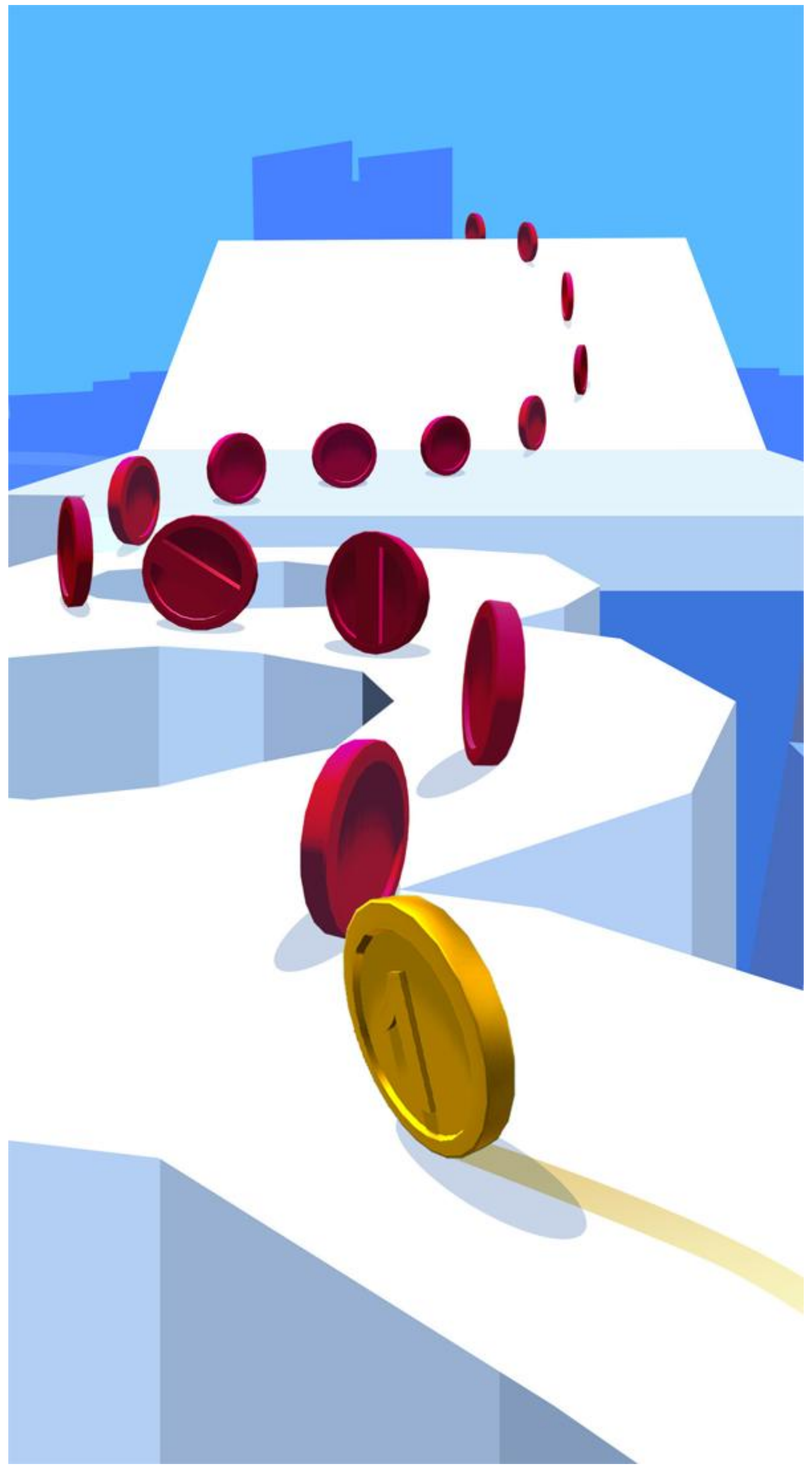
	bias	variance
Constant baseline $$\nabla_\theta J_\theta \approx (R_{\geq t} - b) \nabla_\theta \log \pi_\theta(a_t \mid s_t)$$	none	high one gradient per trajectory
State-based baseline (MC) $$\nabla_\theta J_\theta \approx (R_{\geq t} - V_\phi(s_t)) \nabla_\theta \log \pi_\theta(a_t \mid s_t)$$	none	mid state-dependent baseline
State-based baseline (TD) $$\nabla_\theta J_\theta \approx (r_t + \gamma V_\phi(s_{t+1}) - V_\phi(s_t)) \nabla_\theta \log \pi_\theta(a_t \mid s_t)$$	some V_ϕ is approximate	lower

Multi-step TD

- 1-step TD: $A_t^1 = r_t + \gamma V(s_{t+1}) - V(s_t)$
- 2-step TD: $A_t^2 = r_t + \gamma r_{t+1} + \gamma^2 V(s_{t+2}) - V(s_t)$

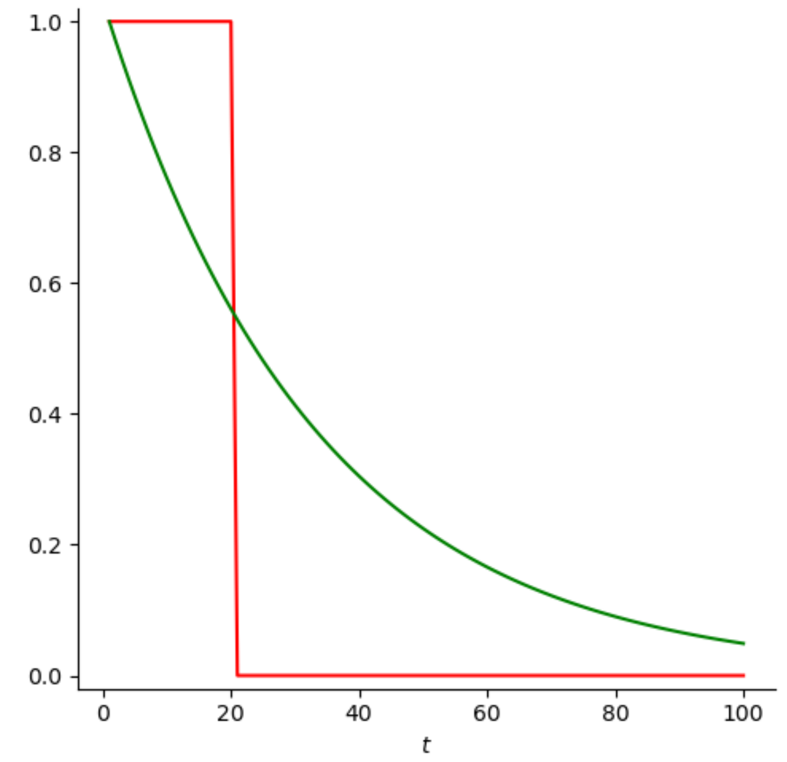
  ...

- n-step TD: $A_t^n = r_t + \cdots + \gamma^{n-1} r_{t+n-1} + \gamma^n V(s_{t+n}) - V(s_t)$
- In the limit (MC): $A_t^\infty = -V(s_t) + r_t + \gamma r_{t+1} + \cdots$



TD(λ)

- How to choose n?
  - Any specific n is hard truncation of the window of evidence we consider
- Instead, use exponential window
  - Take n-step TD with weight proportional to λ^n, where $0 \leq \lambda \leq 1$



$$A_t^\lambda = (1-\lambda)\sum_n \lambda^{n-1} A_t^n = \sum_{\Delta t} (\lambda\gamma)^{\Delta t} \underbrace{(r_{t+\Delta t} + \gamma V(s_{t+\Delta t+1}) - V(s_{t+\Delta t}))}_{A_{t+\Delta t}^1}$$

- Generalized Advantage Estimation (GAE(λ)): $\nabla_\theta J_\theta \approx A_t^\lambda \nabla_\theta \log \pi_\theta(a_t | s_t)$
  - GAE(1) = MC; GAE(0) = 1-step

[Schulman et al., 2015]

Recap

- Policy Gradient = take the gradient of our objective w.r.t. policy parameters
  - Model-free, but on-policy and high variance
- Variance reduction:
  - Past rewards are independent of future actions
  - TD value estimation
  - Baselines, possibly state-dependent
  - TD(λ) to trade off bias and variance

Today's lecture

Advantage estimation

Convergence of RL

Continuous action spaces

Trust-region methods

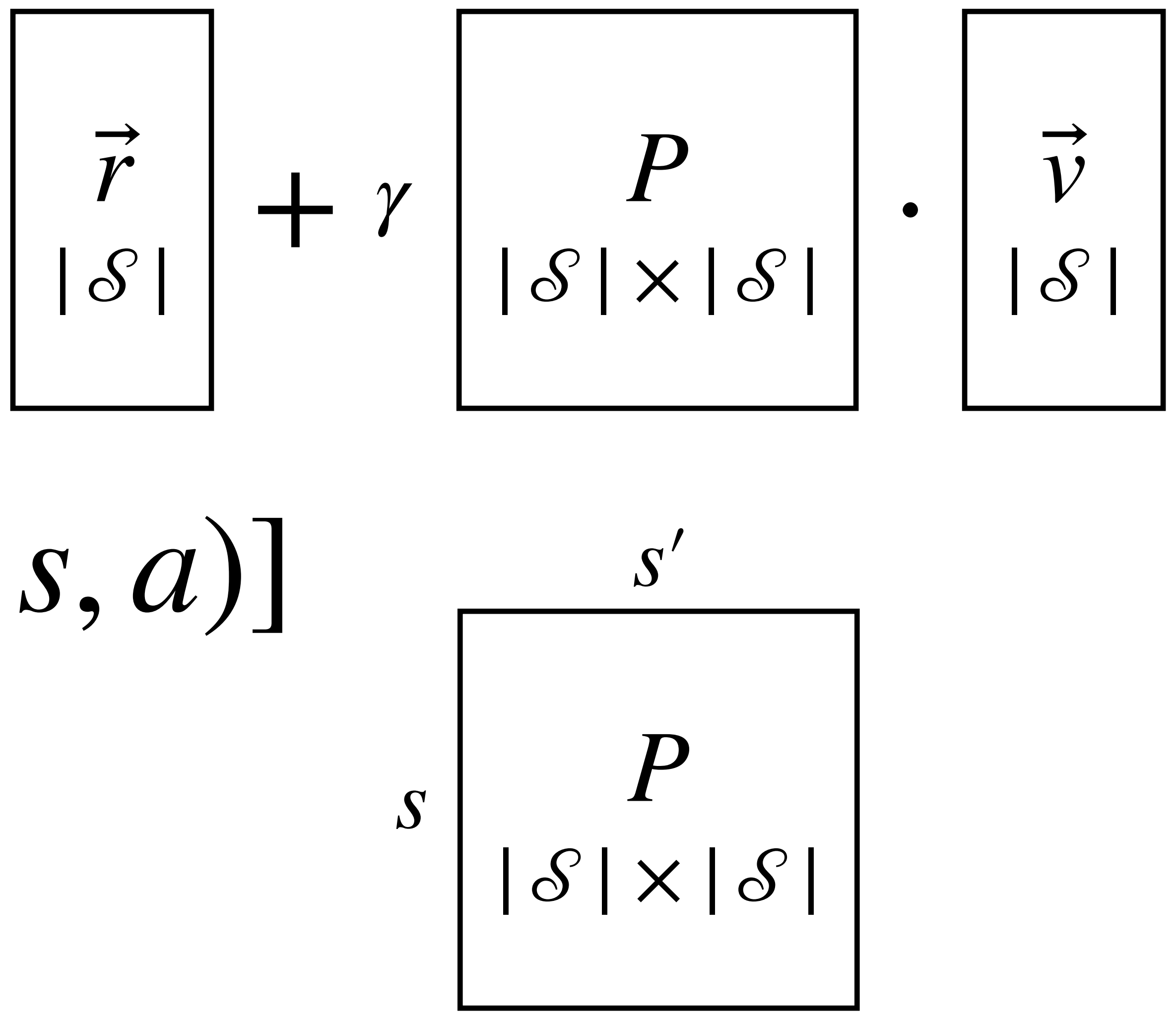
Backup operator

- Value recursion: $V_\pi(s) = \mathbb{E}_{(a|s)\sim\pi}[r(s,a) + \gamma \mathbb{E}_{(s'|s,a)\sim p}[V_\pi(s')]] = \mathcal{T}_\pi[V_\pi](s)$

linear backup operator

- In matrix notation:

$$\vec{v}_\pi = \vec{r}_\pi + \gamma P_\pi \vec{v}_\pi$$



  - where $r_\pi(s) = \mathbb{E}_{(a|s)\sim\pi}[r(s,a)]$, $P_\pi(s,s') = \mathbb{E}_{(a|s)\sim\pi}[p(s'|s,a)]$

- Can be solved with linear algebra:

$$\vec{v}_\pi = (I - \gamma P_\pi)^{-1}\vec{r}_\pi$$

largest eigenvalue magnitude

- The inverse always exists for $\gamma < 1$ because P_π has spectral radius 1

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Bellman operator

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- Bellman operator: $\mathcal{T}[V](s) = \max_a r(s,a) + \gamma \mathbb{E}_{(s'|s,a)\sim p}[V(s')]$
  - Action-value version: $\mathcal{T}[Q](s,a) = r(s,a) + \gamma \mathbb{E}_{(s'|s,a)\sim p}[\max_{a'} Q(s',a')]$
- Value Iteration = iteratively apply $\mathcal{T}$
- Why is this guaranteed to converge? $\mathcal{T}$ is a contraction:

$$\|\mathcal{T}[V_1] - \mathcal{T}[V_2]\|_\infty \leq \max_{s,a} \gamma \mathbb{E}_{(s'|s,a)\sim p}[V_1(s') - V_2(s')] \leq \gamma \|V_1(s') - V_2(s')\|_\infty$$

replace $\mathbb{E}_{s'}$ **with** $\max_{s'}$

- $V^* = \mathcal{T}[V^*]$ is the unique fixed point

Q-Learning convergence

- Q-Learning: $Q(s,a) \to_\alpha r + \gamma \max_{a'} Q(s',a')$

in expectation, this is $\mathcal{T}[Q]$

  - In iteration i, use learning rate α_i

- Robbins–Monro: converges to Q^* with probability 1 (almost surely) if:
  - $\sum_i \alpha_i^2 < \infty$, implying $\alpha_i \to 0$ faster than $i^{-1/2}$
  - $\sum_i \alpha_i = \infty$, implying $\alpha_i \to 0$ not faster than i^{-1}

- Example: $\alpha_i = i^{-1}$ (like in averaging)

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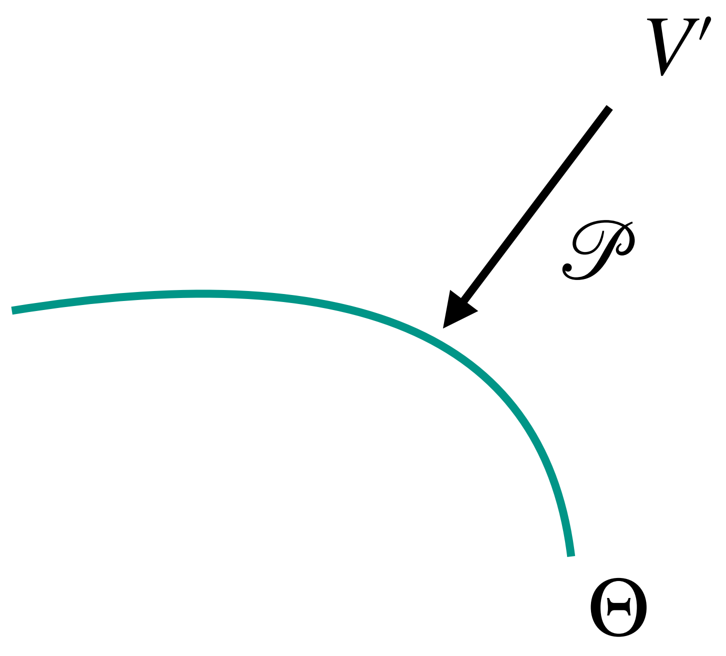
Fitted Value Iteration

- Bellman (TD) error: $\mathscr{T}[V_{\bar\theta}](s) - V_\theta$

- Minimizing the square error is a projection



$$\mathscr{P}[V'] = V_{\arg\min_{\theta\in\Theta} \|V'-V_\theta\|_2^2}$$

- If Θ is convex, the projection is a non-expansion

$$\|\mathscr{P}[V_1'] - \mathscr{P}[V_2']\|_2^2 \le \|V_1' - V_2'\|_2^2$$

- Composition of contractions contracts; but norms mismatch ($\mathscr{T}: L_\infty$; $\mathscr{P}: L_2$)
  - So $\mathscr{P}\mathscr{T}$ is generally not a contraction $\Rightarrow$ no convergence guarantee for FVI

MF | θ | DP | π' | max

But isn't DQN just SGD?

Algorithm DQN

Initialize θ, set $\bar{\theta} \leftarrow \theta$

$s \leftarrow$ reset state

for each interaction step

Sample $a \sim \epsilon$-greedy for $Q_\theta(s, \cdot)$

Get reward r and observe next state s'

Add (s, a, r, s') to replay buffer $\mathcal{D}$

Sample batch $(\vec{s}, \vec{a}, \vec{r}, \vec{s}') \sim \mathcal{D}$

$$y_i \leftarrow \begin{cases} r_i & s'_i \text{ terminal} \\ r_i + \gamma \max_{a'} Q_{\bar{\theta}}(s'_i, a') & \text{otherwise} \end{cases}$$

Descend $\mathcal{L}_\theta = (\vec{y} - Q_\theta(\vec{s}, \vec{a}))^2$

every T_{target} steps, set $\bar{\theta} \leftarrow \theta$

$s \leftarrow$ reset state if s' terminal, else $s \leftarrow s'$

moving target
≠ SGD

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Is PG just SGD?

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max

Algorithm REINFORCE

Initialize π_θ

repeat

Roll out $\xi \sim p_\theta$

Update with gradient $g \leftarrow R(\xi) \sum_t \nabla_\theta \log \pi_\theta(a_t | s_t)$

- The gradient is unbiased for $\nabla_\theta J_\theta$
- The objective J_θ changes with θ, but so does $\mathbb{E}_{x \sim D}[L_\theta(x)]$ in general ML
- But the data distribution changes
- Still, convergence guaranteed as long as we avoid $\pi(a \mid s) = 0$ [Agarwal et al., 2021]

Today's lecture

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Continuous action spaces

Trust-region methods

Continuous actions spaces

can do for large / continuous action spaces?

- What do we need for policy-based methods?
  - For rollouts: given s, sample from $\pi_\theta(a \mid s)$ ✓
  - For policy update: given s and a, compute $\nabla_\theta \log \pi_\theta(a \mid s)$ ✓
- What do we need for value-based methods?
  - For rollouts: given s, compute $\arg\max_a Q_\theta(s, a)$ ✗
  - For value updates: given s, compute $\max_a Q_\theta(s, a)$ ✗
- How can we use value-based methods with continuous action spaces?

Idea 1: DQN with stochastic optimization

- Think of $\max_a Q(s, a)$ as an optimization problem, solved in inner loop
  - Example: stochastic optimization = learn ad-hoc approximately greedy policy π
- Run value-based algorithm; whenever it needs $\max_a Q(s, a)$ search for it:

Algorithm Stochastic optimization

Initialize π
repeat
  Sample $a_1, \ldots, a_k \sim \pi$
  Select k/c top values $Q(s, a_i)$ for $i = 1, \ldots k$
  Fit π to these "elites"

Idea 2: easily maximizable Q

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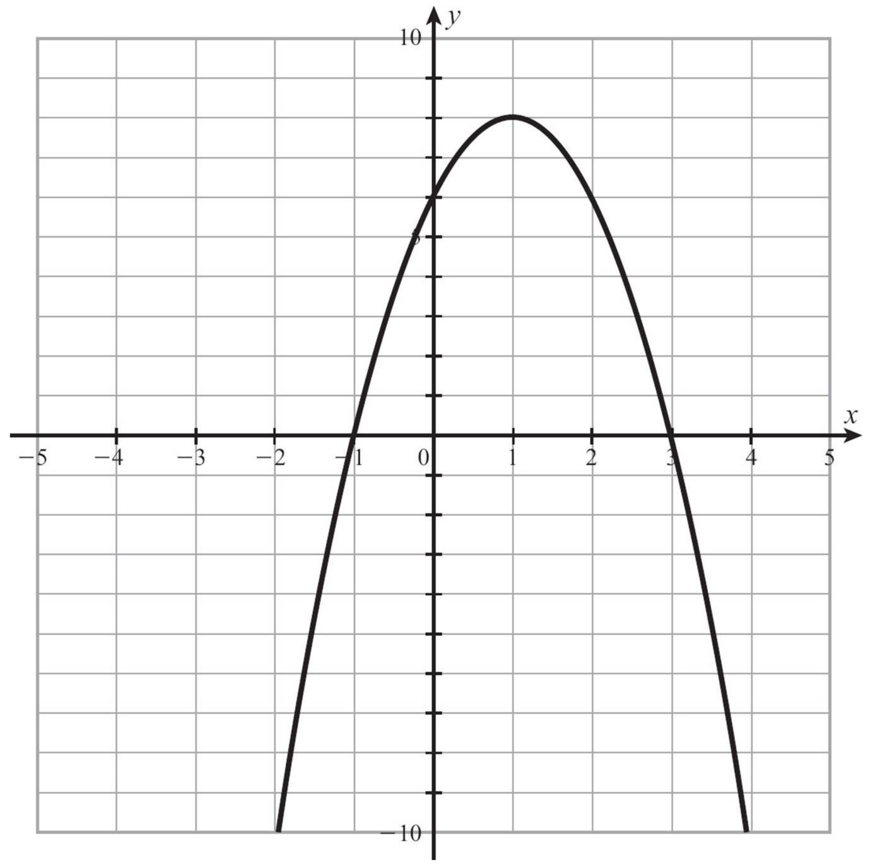
- Represent Q_θ in a way that is directly maximizable

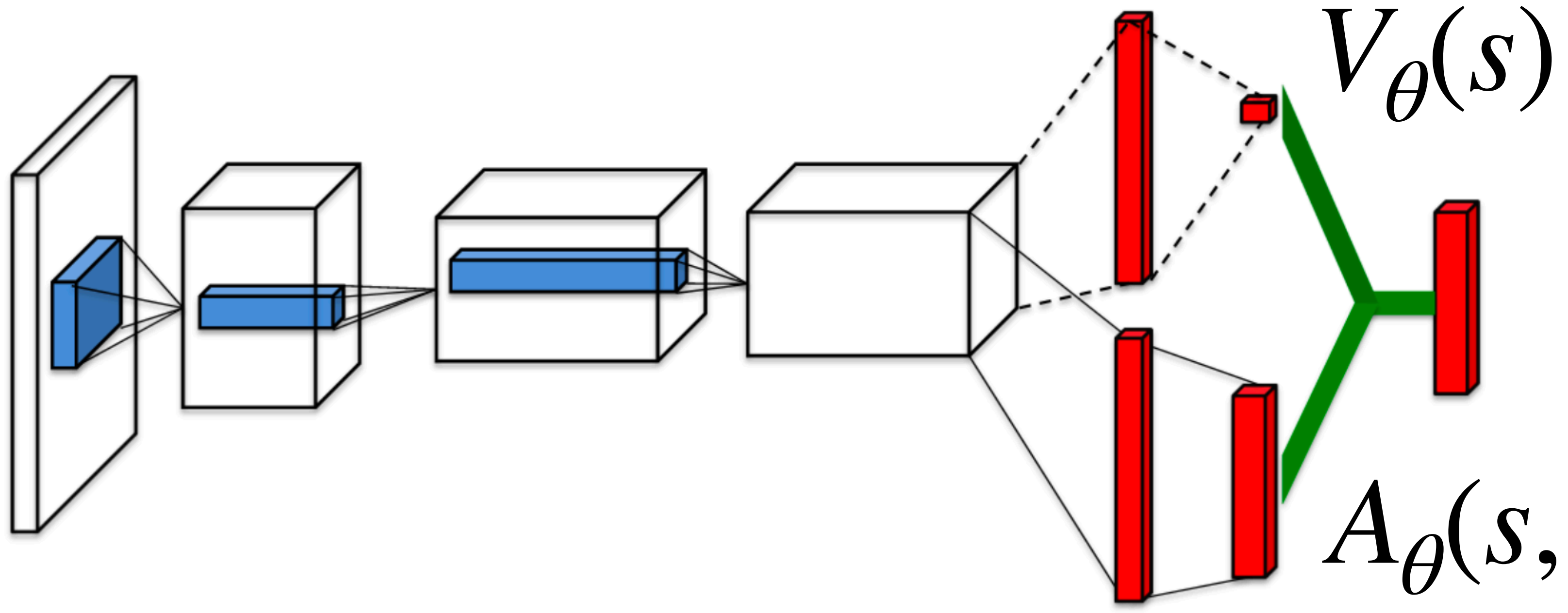
- Example: quadratic $Q_\theta(s,a) = -\frac{1}{2}(a - \mu_\theta(s))^\intercal P_\theta(s)(a - \mu_\theta(s)) + V_\theta(s)$

$$\arg\max_a Q_\theta(s,a) = \mu_\theta(s)$$

$$\max_a Q_\theta(s,a) = V_\theta(s)$$



- Possible architecture: dueling network



[Gu et al., 2016]

Idea 3: learn optimizing policy

- Previous methods: represent a Q maximizer or train one ad-hoc
- More general method: let a deterministic $\mu_\theta(s)$ learn to maximize $Q_\phi(s,a)$
  - This makes it an Actor–Critic method
- Deterministic Policy Gradient Theorem:

$$\nabla_\theta V_{\mu_\theta}(s) = \nabla_\theta Q_{\mu_\theta}(s, \mu_\theta(s)) = \nabla_\theta Q_{\mu_{\bar\theta}}(s, \mu_\theta(s)) + \nabla_\theta Q_{\mu_\theta}(s, \mu_{\bar\theta}(s))$$

$$= \nabla_\theta Q_{\mu_{\bar\theta}}(s, \mu_\theta(s)) + \gamma \mathbb{E}_{(s'|s,\mu_\theta(s))\sim p}[\nabla_\theta V_{\mu_\theta}(s')]$$

pseudo-reward

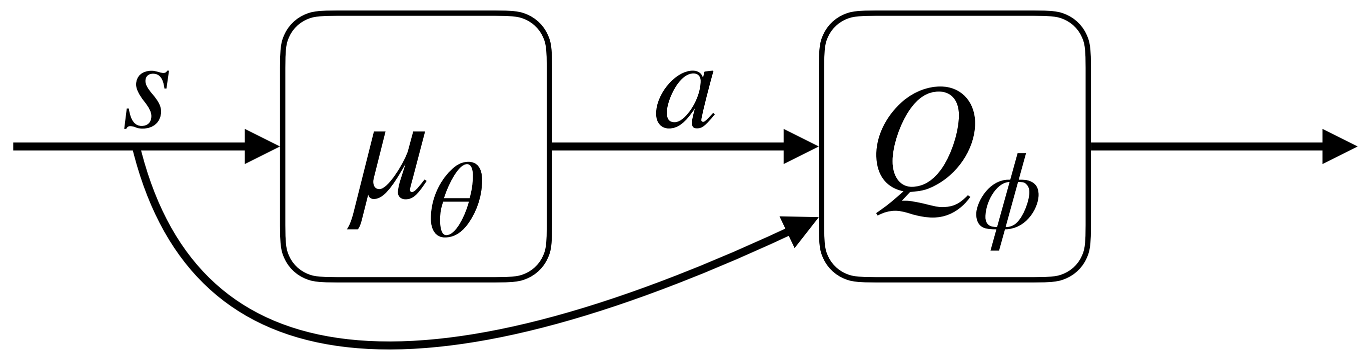
$$\nabla_\theta J_\theta = \sum_t \gamma^t \mathbb{E}_{s_t\sim p_\theta}[\nabla_\theta Q_{\mu_{\bar\theta}}(s_t, \mu_\theta(s_t))] = \cancel{\frac{1}{1-\gamma}} \mathbb{E}_{s\sim p_\theta}[\nabla_\theta Q_{\mu_{\bar\theta}}(s, \mu_\theta(s))]$$

$t \sim \mathrm{Geo}(1-\gamma)$

[Silver et al., 2014]

Deep Deterministic Policy Gradient (DDPG)

- Evaluating Q: feed actor $\mu_\theta(s)$ into critic $Q_\phi(s, a)$



- Back-propagation (chain rule):

$\leftarrow \nabla_\theta \mu_\theta(s) \quad \nabla_a Q_\phi(s, a) \leftarrow$

$$\nabla_\theta J_\theta = \mathbb{E}_{s \sim p_\theta}[\nabla_\theta Q_\phi(s, \mu_\theta(s))]$$
$$= \mathbb{E}_{s \sim p_\theta}[\nabla_\theta \mu_\theta(s) \nabla_a Q_\phi(s, a = \mu_\theta(s))]$$

- DDPG:
  - Train critic Q_ϕ: TD policy evaluation
  - Train actor π_θ: ascend $Q_\phi(s, \mu_\theta)$ with gradient through μ_θ

MF
θ
DP
π'
max

[Lillicrap et al., 2016]

Today's lecture

Advantage estimation

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Importance Sampling

- Suppose you want to estimate $\mathbb{E}_{x\sim p}[f(x)]$
  - but only have samples $x \sim p'$

- Importance sampling:

$$\mathbb{E}_{x\sim p}[f(x)] = \mathbb{E}_{x\sim p'}\left[\frac{p(x)}{p'(x)} f(x)\right]$$

  - Importance (IS) weights: $\rho(x) = \frac{p(x)}{p'(x)}$
  - Estimate: $\rho(x)f(x)$ with $x \sim p'$

IS application 1: multi-step Q-Learning

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- n-step Q-Learning: $Q(s_t, a_t) \to \sum_{\Delta t=0}^{n-1} \gamma^{\Delta t} r_{t+\Delta t} + \gamma^n \max_a Q(s_{t+n}, a)$

- Reminder: $Q^*(s_t, a_t)$ evaluates any a_t but optimal behavior afterward
  - We need data from $a_{t+\Delta t} = \arg\max_a Q(s_{t+\Delta t}, a)$ for RHS to estimate optimal target

- To be off-policy: update $Q(s_t, a_t) \to \sum_{\Delta t=0}^{n-1} \gamma^{\Delta t} \rho_t^{\Delta t} r_{t+\Delta t} + \gamma^n \max_a Q(s_{t+n}, a)$
  - with $\rho_t^{\Delta t} = \prod_{i=t+1}^{t+\Delta t} \frac{\pi(a_i \mid s_i)}{\pi'(a_i \mid s_i)}$ for data from π'

IS application 2: off-policy policy evaluation

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- Estimate $J_\pi = \mathbb{E}_{\xi \sim p_\pi}[R(\xi)]$ off-policy: $J_\pi = \mathbb{E}_{\xi \sim p_{\pi'}}[\rho^\pi_{\pi'}(\xi) R(\xi)]$
  - with $\rho^\pi_{\pi'}(\xi) = \dfrac{p_\pi(\xi)}{p_{\pi'}(\xi)} = \prod_t \dfrac{\pi(a_t|s_t)}{\pi'(a_t|s_t)}$ ← $p(s'|s,a)$ **cancels out**
- $\rho(\xi)$ can be very large or small ⇒ high variance
- Some reduction: r_t is not affected by future actions

$$J_\pi = \sum_t \mathbb{E}_{\xi_{\leq t} \sim p_{\pi'}}[\gamma^t \rho^\pi_{\pi'}(\xi_{\leq t}) r_t] = \sum_t \mathbb{E}_{\xi_{\leq t} \sim p_{\pi'}}\left[\gamma^t r_t \prod_{t' \leq t} \frac{\pi(a_{t'}|s_{t'})}{\pi'(a_{t'}|s_{t'})}\right]$$

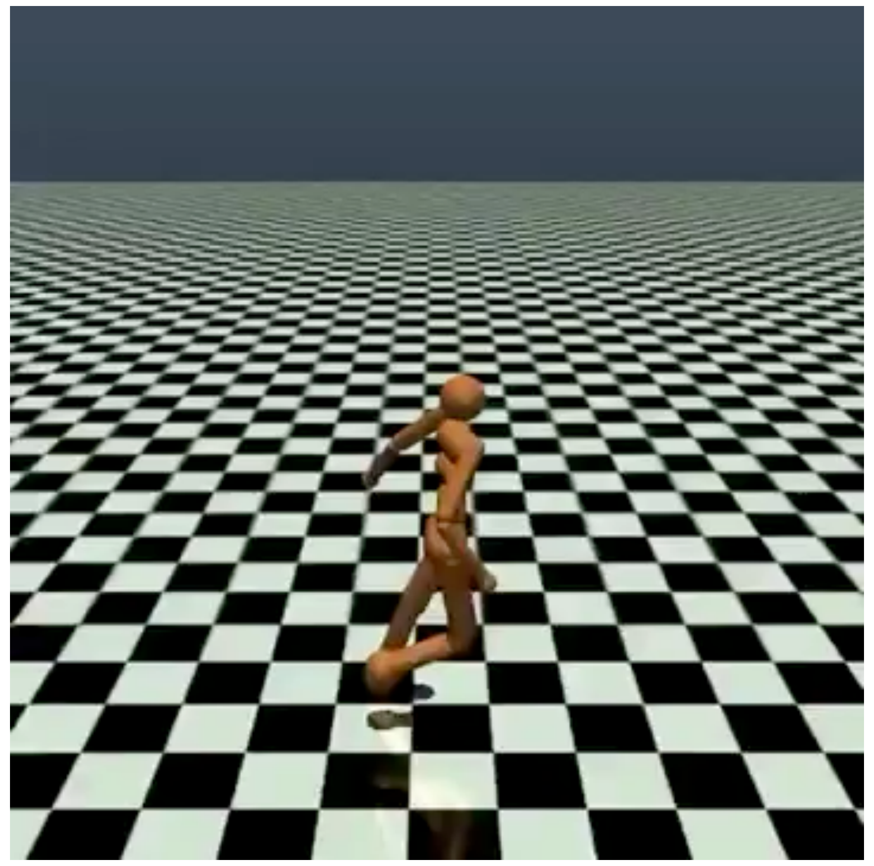
[Precup et al., 2000]

IS application 3: Off-policy Policy Gradient

- Policy Gradient: $\nabla_\theta J_\theta = \sum_t \gamma^t \mathbb{E}_{\xi \sim p_\theta}[R_{\geq t}(\xi) \nabla_\theta \log \pi_\theta(a_t | s_t)]$

- Off-Policy PG: $\nabla_\theta J_\theta = \sum_t \gamma^t \mathbb{E}_{\xi \sim p_{\theta'}}[\rho_{\theta'}^{\theta}(\xi_{\leq t}) R_{\geq t}(\xi) \nabla_\theta \log \pi_\theta(a_t | s_t)]$

  - $R_{\geq t}(\xi)$ = future discounted rewards affected by $\pi_\theta(a_t | s_t)$
  - $\rho_{\theta'}^{\theta}(\xi_{\leq t})$ = past probability ratios that affect $\pi_\theta(a_t | s_t)$



- Should we discount by γ^t? Not if we care about evidence from later states

- $\rho_{\theta'}^{\theta}(\xi_{\leq t})$ has high variance, some methods just use $\rho_{\theta'}^{\theta}(a_t | s_t) = \dfrac{\pi_\theta(a_t | s_t)}{\pi_{\theta'}(a_t | s_t)}$

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[Liu et al., 2018]

Performance Difference Lemma

- Policy gradient = small changes in policy; can we make large changes?

telescopic cancelation

- For any π, ξ: $\sum_t \gamma^t A_\pi(s_t, a_t) = \sum_t \gamma^t (r_t + \gamma V_\pi(s_{t+1}) - V_\pi(s_t)) = R(\xi) - V_\pi(s_0)$

advantage of entire trajectory

- Expectation by different policy: Performance Difference Lemma

$$\sum_t \gamma^t \mathbb{E}_{(s_t,a_t)\sim p_\pi}[A_{\bar{\pi}}(s_t, a_t)] = \mathbb{E}_{\xi\sim p_\pi}[R(\xi) - V_{\bar{\pi}}(s_0)] = J_\pi - J_{\bar{\pi}}$$

$s_0 \sim p$ **in both** π **and** π'

  - We want to maximize over π, with $\bar{\pi}$ fixed

- Compare: PG Theorem $\nabla_\theta J_\theta = \sum_t \gamma^t \mathbb{E}_{(s_t,a_t)\sim p_\theta}[A_{\pi_\theta}(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t | s_t)]$

[Kakade and Langford, 2002]

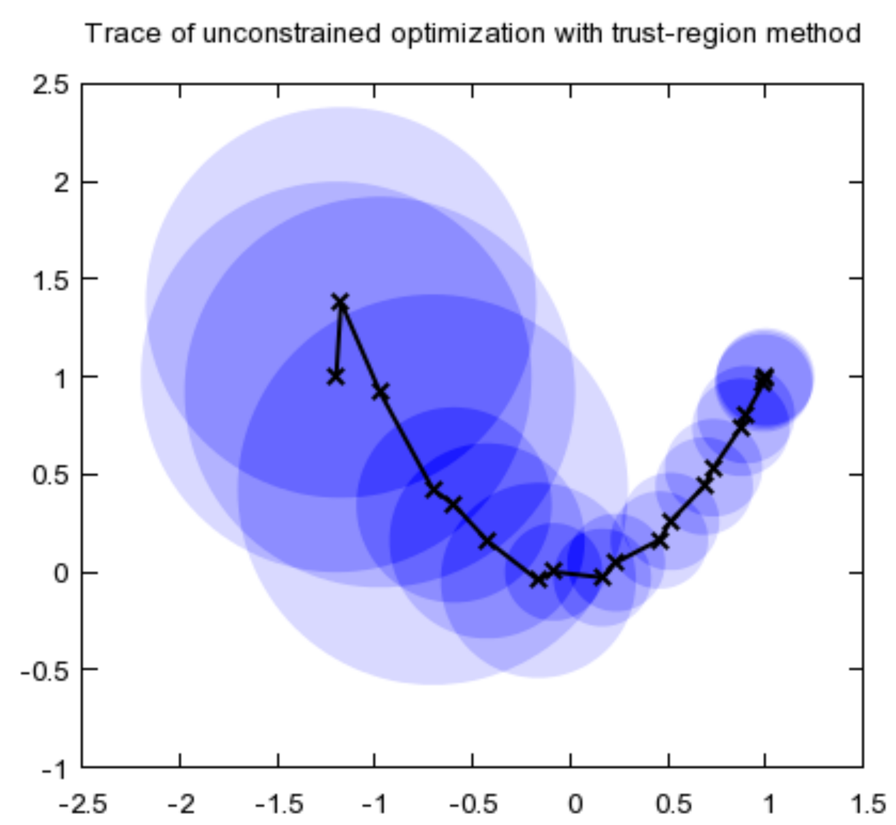
Finding best next policy

- With current policy $\bar{\pi}$: find $\max_\pi J_\pi - J_{\bar{\pi}} = \max_\pi \sum_t \gamma^t \mathbb{E}_{(s_t,a_t)\sim p_\pi}[A_{\bar{\pi}}(s_t, a_t)]$
  - Can use $\bar{\pi}$ to evaluate $A_{\bar{\pi}}$
- But we don't have data $(s_t, a_t) \sim p_\pi$; idea: sample from $\bar{\pi}$
  - Trick question: is this on-policy or off-policy? On-policy data, but needs IS weight

$$\max_\pi \sum_t \gamma^t \mathbb{E}_{\xi_{\leq t}\sim p_{\bar{\pi}}}[\rho^\pi_{\bar{\pi}}(\xi_{\leq t}) A_{\bar{\pi}}(s_t, a_t)]$$

- Is it reasonable to use $\rho^\pi_{\bar{\pi}}(a_t | s_t) = \dfrac{\pi(a_t | s_t)}{\bar{\pi}(a_t | s_t)}$ instead? i.e. drop $\rho^\pi_{\bar{\pi}}(\xi_{<t})$

Trust-Region Policy Optimization (TRPO)



- Trust region = space around $\bar{\pi}$ where $\rho(\xi_{<t}) \approx 1$
  - Easier to consider $\mathbb{E}_{\xi_{<t} \sim p_{\bar{\pi}}}[\log \rho(\xi_{<t})] \approx 0$

$$-\mathbb{E}_{\xi_{<t} \sim p_{\bar{\pi}}}[\log \rho(\xi_{<t})] = \mathbb{D}[\bar{\pi}(\xi_{<t}) \| \pi(\xi_{<t})] = \sum_{t'<t} \mathbb{E}_{\xi_{<t'} \sim p_{\bar{\pi}}}[\mathbb{D}[\bar{\pi}(a_{t'} | s_{t'}) \| \pi(a_{t'} | s_{t'})]]$$

- TRPO: $\max_\theta \mathbb{E}_{(s,a) \sim p_{\bar{\theta}}}[\rho_{\bar{\theta}}^{\theta}(a | s) A_{\bar{\theta}}(s, a)]$ s.t. $\mathbb{E}_{s \sim p_{\bar{\theta}}}[\mathbb{D}[\pi_{\bar{\theta}}(a | s) \| \pi_\theta(a | s)]] \leq \epsilon$
  - $A_{\bar{\theta}}$ estimated with critic A_ϕ
  - Computational tricks for gradient-based optimization

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[Schulman et al., 2015]

Proximal Policy Optimization (PPO)

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π'

max

- Same motivation: ascend $\mathbb{E}_{(s,a)\sim p_{\bar\theta}}[\rho_{\bar\theta}^{\theta}(a\,|\,s)A_{\bar\theta}(s,a)]$ with π_θ staying near $\pi_{\bar\theta}$
  - PPO-Penalty: add a penalty term for $\mathbb{E}_{s\sim p_{\bar\theta}}[\mathbb{D}[\pi_{\bar\theta}(a\,|\,s)\|\pi_\theta(a\,|\,s)]]$
  - PPO-Clip: ascend $\mathbb{E}_{(s,a)\sim p_{\bar\theta}}[L_{\bar\theta}^{\theta}(s,a)]$ with

$$L_{\bar\theta}^{\theta}(s,a) = \min(\rho_{\bar\theta}^{\theta}(a\,|\,s)A_{\bar\theta}(s,a), A_{\bar\theta}(s,a) + |\,\epsilon A_{\bar\theta}(s,a)\,|)$$

- This caps the incentive for θ to deviate from $\bar\theta$ at:
  - $\rho_{\bar\theta}^{\theta}(a\,|\,s) \le 1+\epsilon$ for $A_{\bar\theta}(s,a) \ge 0$ (when we want to increase $\pi_\theta(a\,|\,s)$)
  - $\rho_{\bar\theta}^{\theta}(a\,|\,s) \ge 1-\epsilon$ for $A_{\bar\theta}(s,a) \le 0$(when we want to decrease $\pi_\theta(a\,|\,s)$)

no incentive ≠ doesn't happen
PPO has lots more tricks
to limit divergence

[Schulman et al., 2017]

Recap

- Model-based policy evaluation can be solved linearly
- Deep RL isn't just SGD
  - Exception: policy gradient on offline (batch) data
- Value-based methods struggle to $\max$ in continuous action spaces
  - DDPG: π_θ learns to maximize Q_ϕ (actor–critic method)
- Importance Sampling decouples expectation and sampling distributions
  - Optimize on-policy objectives with off-policy data
  - TRPO and PPO: sample from current policy to evaluate next policy, if it's close