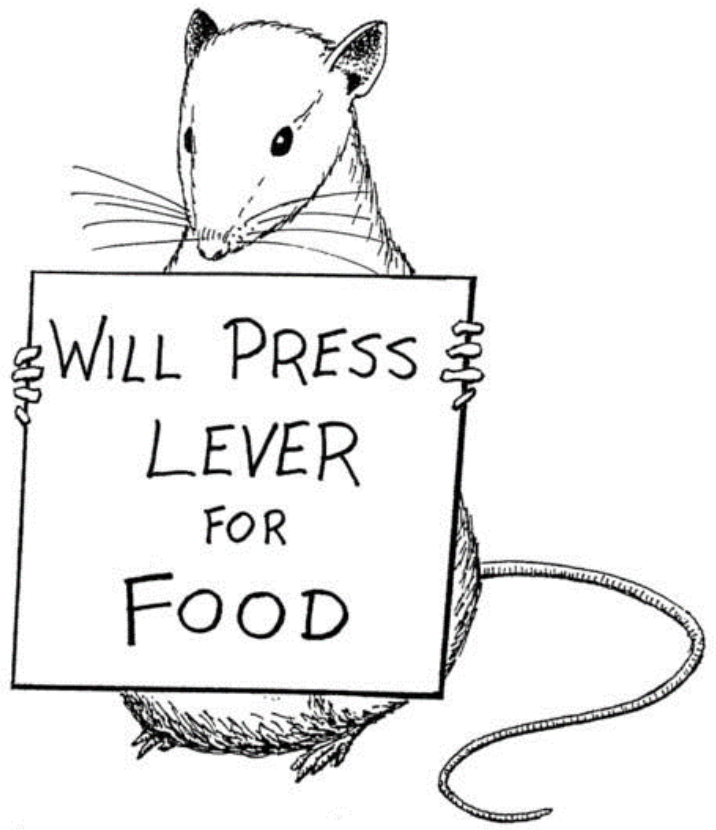
# CS 277: Control and Reinforcement Learning

Winter 2024

# Lecture 4: Deep Q-Learning

Roy Fox
Department of Computer Science
School of Information and Computer Sciences
University of California, Irvine



# Logistics

assignments

- Exercise 1 due tomorrow (or Sunday)
- Quiz 2 due next Monday

office hours

- Fixed hours starting next week
- Contact me for special accommodation
- Please keep using this resource!

# Today's lecture

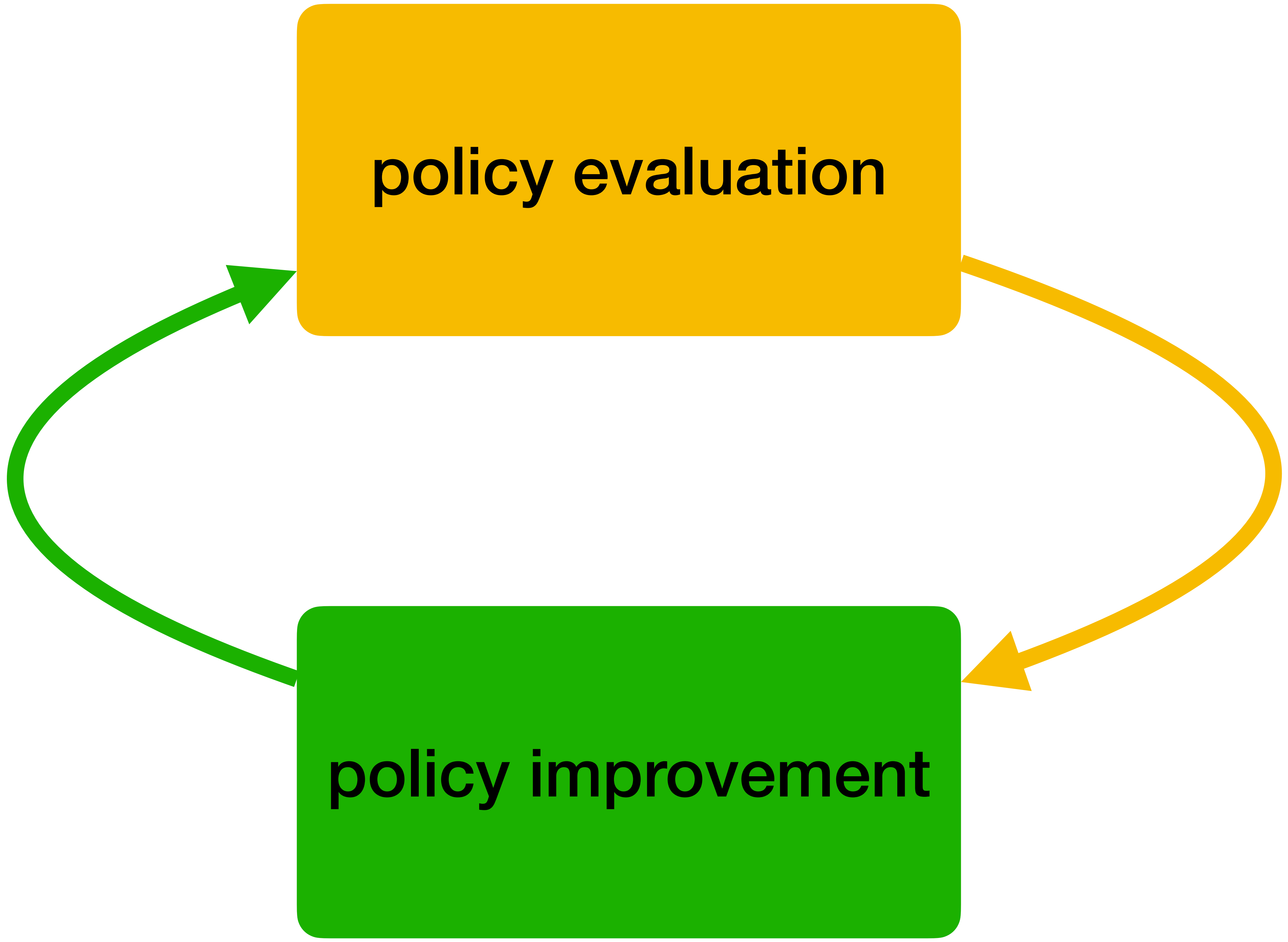
Policy Improvement

Fitted Q-Iteration

Deep Q-Learning

DQN tricks

# The RL scheme



# Policy improvement

- A value function suggests the greedy policy:

$$\pi(s) = \arg\max_a Q(s,a) = \arg\max_a (r(s,a) + \gamma \mathbb{E}_{(s'|s,a)\sim p}[V(s')])$$

- The greedy policy may not be the optimal policy $\pi^* = \arg\max_\pi J_\pi$
  - But is the greedy policy always an improvement?
- Proposition: the greedy policy for $Q_\pi$ (value of $\pi$) is never worse than $\pi$
- Corollary (Bellman optimality): if $\pi$ is greedy for its value $Q_\pi$ then it is optimal
  - In a finite MDP, the iteration $\pi \xrightarrow{\text{evaluate}} Q_\pi \xrightarrow{\text{greedy}} \pi$ converges, and then $\pi$ is optimal

# Policy Iteration



- If we know the MDP (model-based), we can just alternate evaluate/greedy:

**Algorithm** Policy Iteration

Initialize some policy $\pi$
**repeat**
  Evaluate the policy $Q(s, a) \leftarrow \mathbb{E}_{\xi \sim p_\pi}[R | s_0 = s, a_0 = a]$
  Update to the greedy policy $\pi(s) \leftarrow \arg\max_a Q(s, a)$

- Upon convergence, $\pi = \pi^*$ and $Q = Q^*$

# Value Iteration

- We can also alternate evaluate/greedy inside the loop over states:

**Algorithm** Value Iteration

Initialize some value function $V$
**repeat**
  **for each** state $s$
    Update $V(s) \leftarrow \max_a (r(s,a) + \gamma\, \mathbb{E}_{(s'|s,a)\sim p}[V(s')])$

- Must update each state repeatedly until convergence

- Upon convergence, $\pi^*(s) = \arg\max_a (r(s,a) + \gamma \mathbb{E}_{(s'|s,a)\sim p}[V(s')])$

MF | $\theta$ | DP | $\pi'$ | max

# Generalized Policy Iteration

MF

$\theta$

DP

$\pi'$

max

- We can even alternate in any order we wish:

$$V(s) \leftarrow \mathbb{E}_{(a|s)\sim\pi}[r(s,a) + \gamma\mathbb{E}_{(s'|s,a)\sim p}[V(s')]]$$

$$\pi(s) \leftarrow \arg\max_a(r(s,a) + \gamma\mathbb{E}_{(s'|s,a)\sim p}[V(s')])$$

- As long as each state gets each of the two update without starvation
  - The process will eventually converge to $V^*$ and $\pi^*$

# Model-free reinforcement learning

- We can be model-free using MC policy evaluation:

MF
$\theta$
DP
$\pi'$
max

**Algorithm** MC model-free RL

Initialize some policy $\pi$
**repeat**
  Initialize some value function $Q$
  **repeat** to convergence
    Sample $\xi \sim p_\pi$
    Update $Q(s_t, a_t) \to R_{\geq t}(\xi)$ for all $t \geq 0$
  $\pi(s) \leftarrow \arg\max_a Q(s, a)$ for all $s$

- On-policy policy evaluation in the inner loop — very inefficient

# Off-policy model-free reinforcement learning

- Value iteration is model-based: $V(s) \leftarrow \max_a (r(s,a) + \gamma \mathbb{E}_{(s'|s,a)\sim p}[V(s')])$
- Action-value version: $Q(s,a) \leftarrow r(s,a) + \gamma \mathbb{E}_{(s'|s,a)\sim p}[\max_{a'} Q(s',a')]$
- A model-free (data-driven) version — Q-Learning:
  - On off-policy data $(s,a,r,s')$, update

$$Q(s,a) \to r + \gamma \max_{a'} Q(s',a')$$

MF

$\theta$

DP

$\pi'$

max

[Watkins and Dayan, 1992]

# Recap

- RL is a (policy evaluation ↔ policy improvement) loop
- Policy evaluation: model-based, Monte Carlo, or Temporal-Difference
  - Temporal-Difference exploits the sequential structure using dynamic programming
- TD can be off-policy by considering the action-value Q function
  - Off-policy data can be thrown out less often as the policy changes
- Policy improvement can be greedy
  - Arbitrarily alternated with policy evaluation of any kind (MB, MC, or TD)
- Many approaches can be made differentiable for Deep RL

# Today's lecture

Policy Improvement

Fitted Q-Iteration

Deep Q-Learning

DQN tricks

# Fitted Value-Iteration (FVI)

**Algorithm** Value Iteration

Initialize some value function $V$
**repeat**
  **for each** state $s$
    Update $V(s) \leftarrow \max_a (r(s,a) + \gamma\, \mathbb{E}_{(s'|s,a)\sim p}[V(s')])$

MF θ DP π′ max

- Fitted Value-Iteration (FVI):

square error

$$\theta^{i+1} \leftarrow \arg\min_\theta \mathbb{E}_{s\sim\mu}[(\max_a(r(s,a) + \gamma \mathbb{E}_{(s'|s,a)\sim p}[V_{\theta^i}(s')]) - V_\theta(s))^2]$$

MF θ DP π′ max

  - For some state distribution $\mu$
  - Can use losses other than square

# Fitted Q-Iteration (FQI)

- Action-value iteration: $Q(s,a) \leftarrow r(s,a) + \gamma \mathbb{E}_{(s'|s,a)\sim p}[\max_{a'} Q(s',a')]$

- Fitted Q-Iteration (FQI):

$$\theta^{i+1} \leftarrow \arg\min_{\theta} \mathbb{E}_{(s,a)\sim\mu}[(r(s,a) + \gamma \mathbb{E}_{(s'|s,a)\sim p}[\max_{a'} Q_{\theta^i}(s',a')]) - Q_\theta(s,a))^2]$$

  - For some state-action distribution $\mu$

MF $\theta$ DP $\pi'$ max

- We can also combine
  - Policy evaluation: MC with function approximation
  - Policy improvement: greedy

MF $\theta$ DP $\pi'$ max

# Q-Learning

MF

$\theta$

DP

$\pi'$

max

**Algorithm** Q-Learning

Initialize $Q$

$s \leftarrow$ reset state

**repeat**

Take some action $a$

Receive reward $r$

Observe next state $s'$

Update $Q(s,a) \rightarrow \begin{cases} r & s' \text{ terminal} \\ r + \gamma \max_{a'} Q(s',a') & \text{otherwise} \end{cases}$

$s \leftarrow$ reset state if $s'$ terminal, else $s \leftarrow s'$

[Watkins and Dayan, 1992]

# Sampling-based Fitted Q-Iteration

- FQI can be model-free by sampling from $p$
  - We can sample using environment interaction with some $\pi'$, if $\mu = p_{\pi'}$
  - Or sample using a simulator we can reset to any state $s \sim \mu$
  - Anyway, this is off-policy from the greedy policy $\arg\max_a Q_\theta(s, a)$

MF

$\theta$

DP

$\pi'$

max

**Algorithm** Sampling-based Fitted Q-Iteration

Initialize $\theta$

**repeat**

Sample a batch $(\vec{s}, \vec{a}) \sim \mu$

Feed to simulator to get batch $(\vec{r}, \vec{s}')$

Descend $\mathcal{L}_\theta = (\vec{r} + \gamma \max_{\vec{a}'} Q_{\bar{\theta}}(\vec{s}', \vec{a}') - Q_\theta(\vec{s}, \vec{a}))^2$

[Munos and Szepesvári, 2008]

# Today's lecture

Policy Improvement

Fitted Q-Iteration

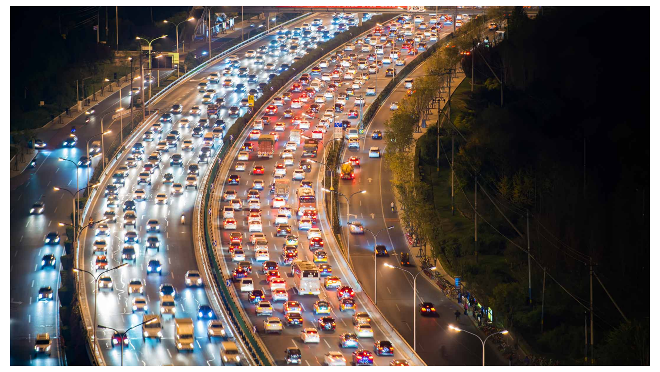
**Deep Q-Learning**

DQN tricks

# Experience policy

- Which distribution should the training data have?
  - The policy may not be good on other distributions / unsupported states
  - $\Rightarrow$ ideally, the test distribution $p_\pi$ for the final $\pi$
- On-policy methods (e.g. MC): must use on-policy data (from the current $\pi$)
- Off-policy methods (e.g. Q) can use different policy (or even non-trajectories)
  - But both should eventually use $p_\pi$ or suffer train–test distribution mismatch

# Exploration policies



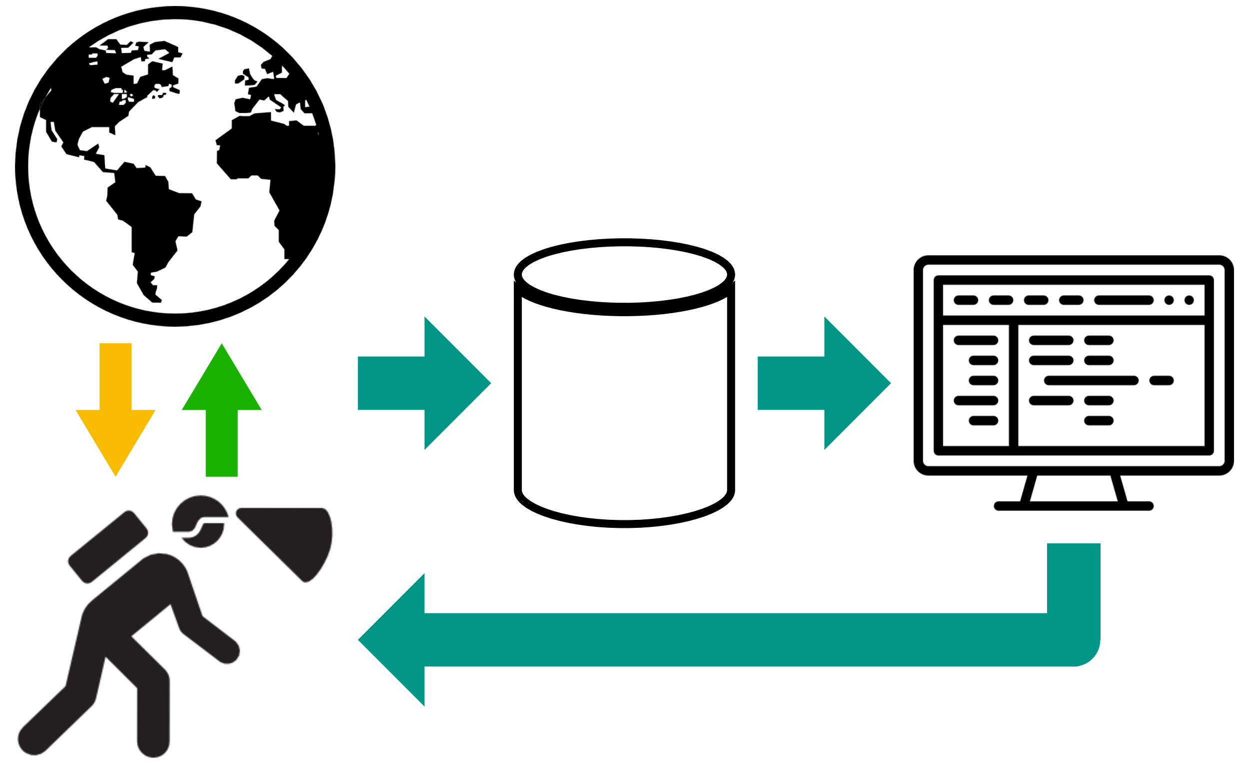
- Example: I tried route 1: {40, 20, 30}; route 2: {30, 25, 40}
  - Suppose route 1 really has expected time 30min, should you commit to it forever?
- To avoid overfitting, we must try all actions infinitely often
- $\epsilon$-greedy exploration: select uniform action with prob. $\epsilon$, otherwise greedy
- Boltzmann exploration:

$$\pi(a|s) = \operatorname*{soft\,max}_a(Q(s,a);\beta) = \frac{\exp(\beta Q(s,a))}{\sum_{\bar{a}} \exp(\beta Q(s,\bar{a}))}$$

  - Becomes uniform as the inverse temperature $\beta \to 0$, greedy as $\beta \to \infty$

# Experience replay

- On-policy methods are inefficient: throw out all data with each policy update
- Off-policy methods can keep the data = experience replay
  - Replay buffer: dataset of past experience
  - Diversifies the experience (beyond current trajectory)
- Variants differ on
  - How often to add data vs. sample data
  - How to sample from the buffer
  - When to evict data from the buffer, and which

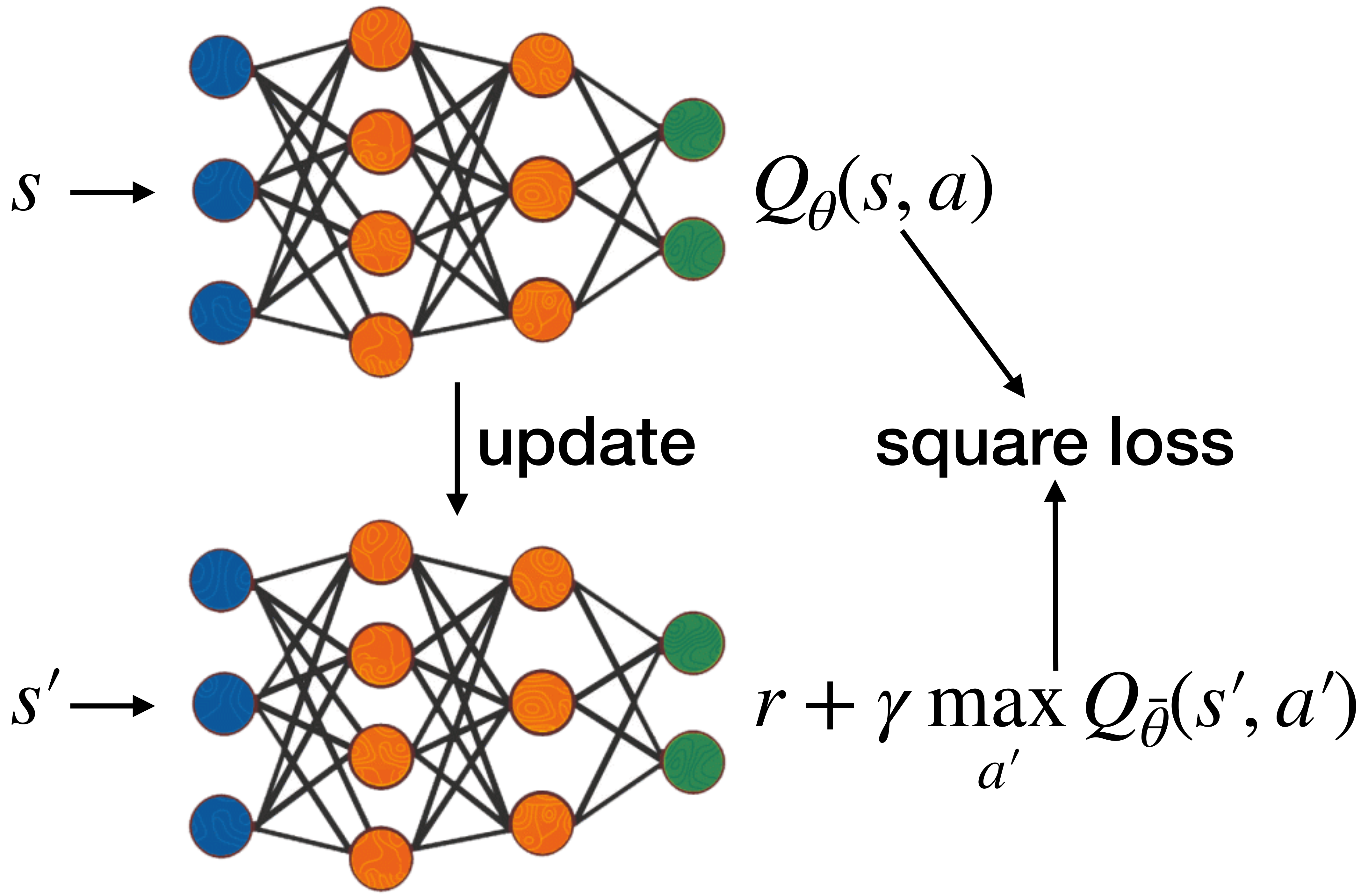


# Why use target network?

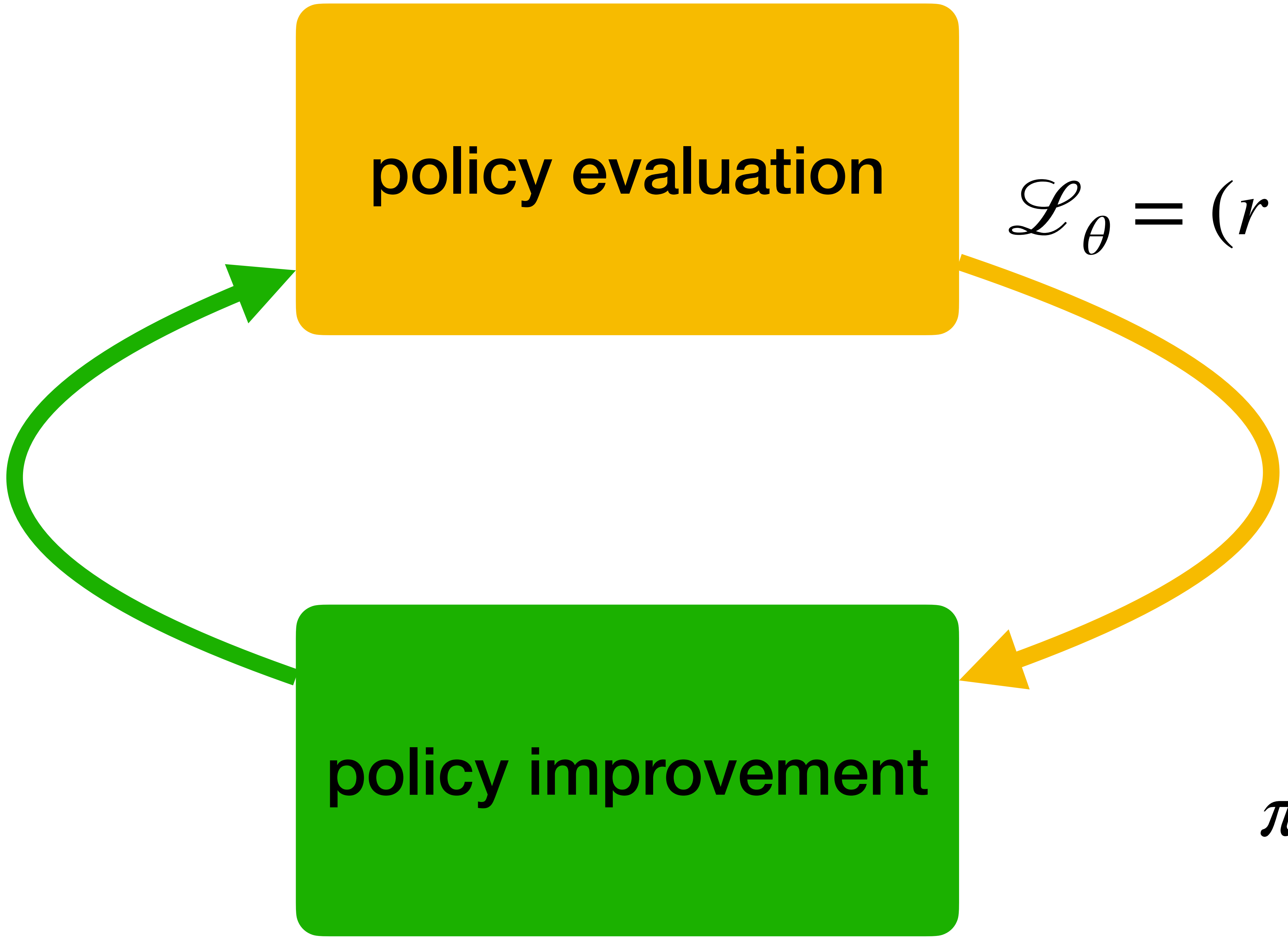
- Fitted-Q loss: $\mathcal{L}_\theta = (r + \gamma \max_{a'} Q_{\bar{\theta}}(s', a') - Q_\theta(s, a))^2$

  no gradient from the target term

- Target network = lagging copy of $Q_\theta(s, a)$
  - Periodic update: $\bar{\theta} \leftarrow \theta$ every $T_{\text{target}}$ steps
  - Exponential update: $\bar{\theta} \leftarrow (1 - \eta)\bar{\theta} + \eta\theta$
- $Q_{\bar{\theta}}$ is more stable
  - Less of a moving target
  - Less sensitive to data ⇒ less variance
- But $\bar{\theta} \neq \theta$ introduces bias



# Putting it all together: DQN




**fitted-Q loss**

$$\mathcal{L}_\theta = (r + \gamma \max_{a'} Q_{\bar{\theta}}(s', a') - Q_\theta(s, a))^2$$

**exploration**

**e.g. $\epsilon$-greedy**

**greedy policy**

$$\pi(s) = \arg\max_a Q(s, a)$$

# Deep Q-Learning (DQN)

**Algorithm** DQN

Initialize $\theta$, set $\bar{\theta} \leftarrow \theta$

$s \leftarrow$ reset state

**for each** interaction step

Sample $a \sim \epsilon$-greedy for $Q_\theta(s, \cdot)$

Get reward $r$ and observe next state $s'$

Add $(s, a, r, s')$ to replay buffer $\mathcal{D}$

Sample batch $(\vec{s}, \vec{a}, \vec{r}, \vec{s}') \sim \mathcal{D}$

$$y_i \leftarrow \begin{cases} r_i & s'_i \text{ terminal} \\ r_i + \gamma \max_{a'} Q_{\bar{\theta}}(s'_i, a') & \text{otherwise} \end{cases}$$

Descend $\mathcal{L}_\theta = (\vec{y} - Q_\theta(\vec{s}, \vec{a}))^2$

every $T_{\text{target}}$ steps, set $\bar{\theta} \leftarrow \theta$

$s \leftarrow$ reset state if $s'$ terminal, else $s \leftarrow s'$

MF

$\theta$

DP

$\pi'$

max

[Mnih et al., 2015]

# Today's lecture

Policy Improvement

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Deep Q-Learning

**DQN tricks**

# Value estimation bias

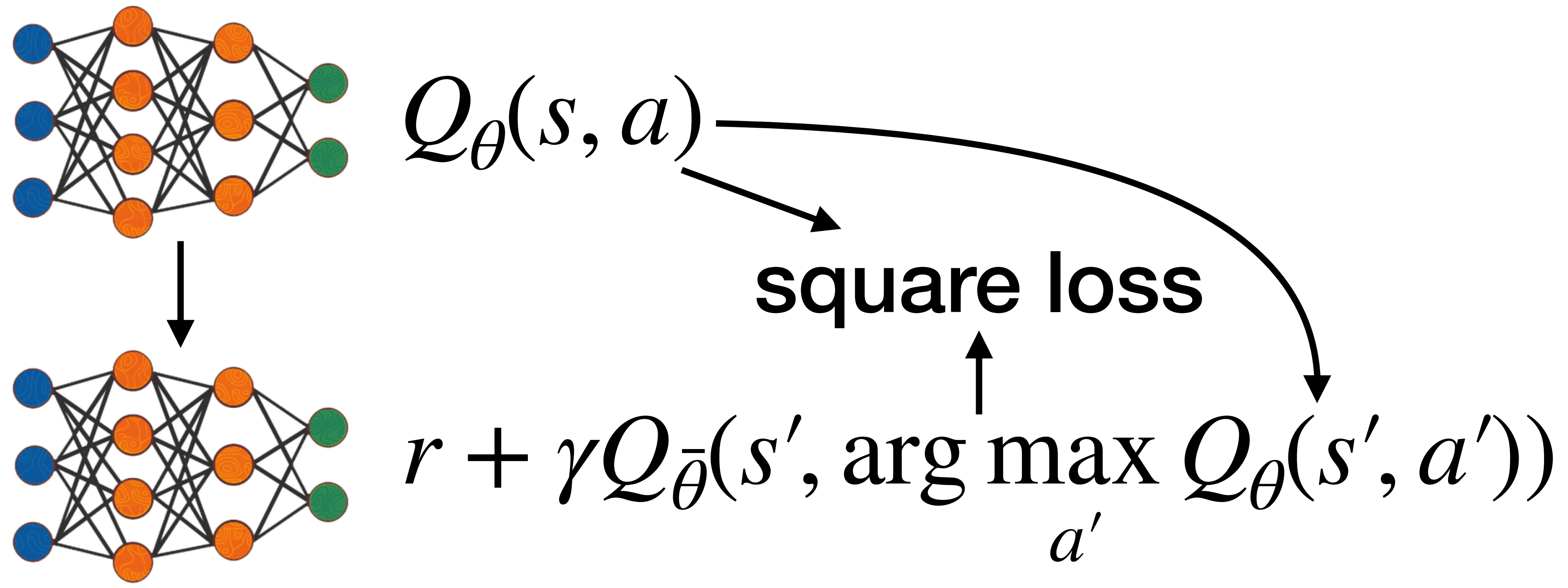
- Q-value estimation is optimistically biased
- Jensen's inequality: for a random vector $Q$

$$\mathbb{E}[\max_a Q_a] \geq \max_a \mathbb{E}[Q_a]$$

- While there's uncertainty in $Q_{\bar{\theta}}$, $\max_{a'} Q_{\bar{\theta}}(s', a')$ is positively biased
- So how can this converge?
  - As certainty increases, the bias of each update decreases
  - Existing bias attenuates with repeated discounting by $\gamma$

# Double Q-Learning

- Idea: keep two value estimates $Q_1$ and $Q_2$
  - Update: $Q_i(s,a) \to r + \gamma Q_{-i}(s', \arg\max_{a'} Q_i(s',a'))$
    
    $-i$ = the other
- How to use this with DQN?
- Idea 1: use target network as the other estimate




- Idea 2: Clipped Double Q-Learning

$$Q_{\theta_i}(s,a) \to r + \gamma \min_{i=1,2} Q_{\bar\theta_i}(s', \arg\max_{a'} Q_{\theta_i}(s',a'))$$

[van Hasselt, 2010]

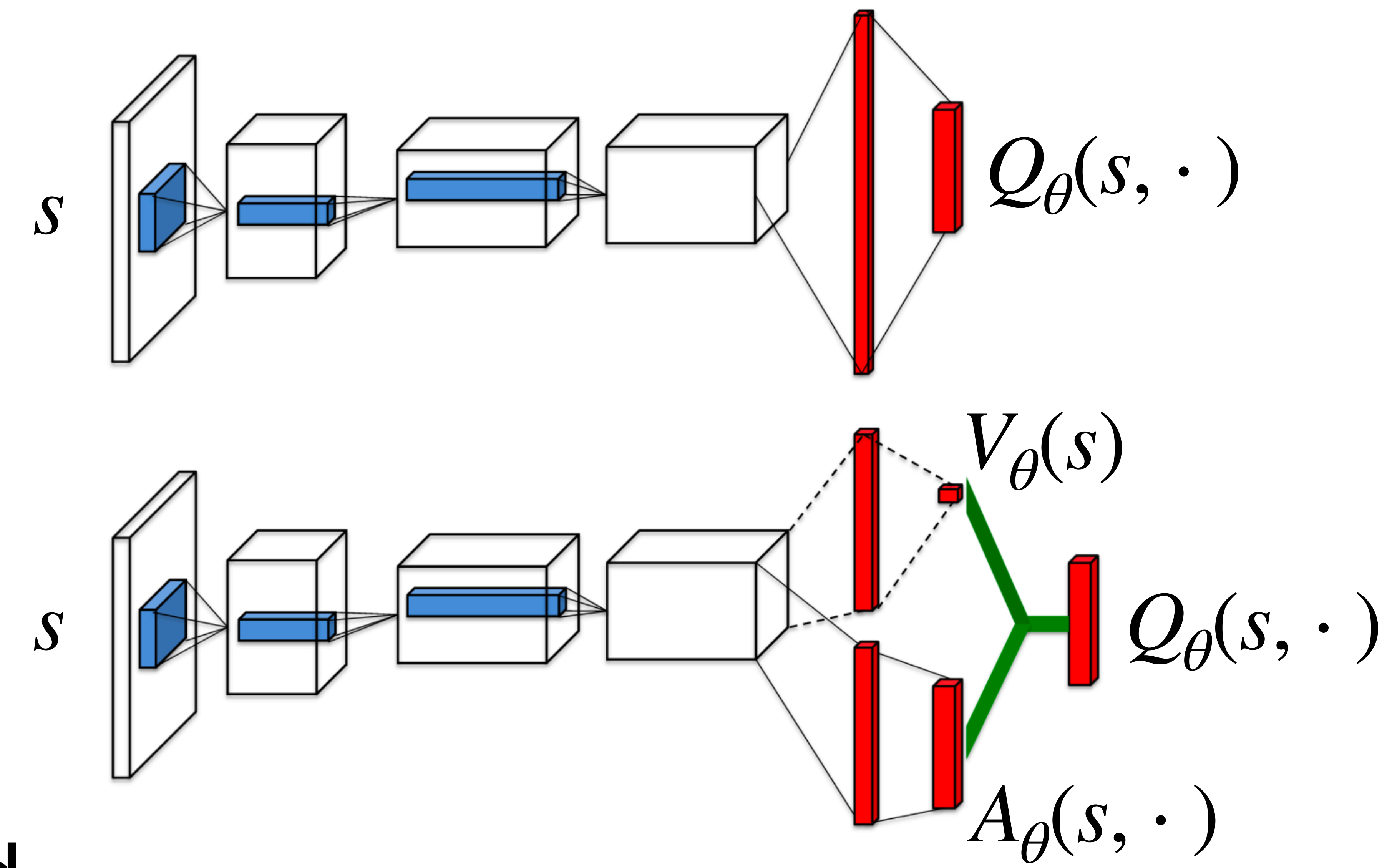
# Prioritized Experience Replay

- Bellman error (= TD error): $\delta(s, a, r, s') = r + \gamma \max_{a'} Q(s', a') - Q(s, a)$
  - Optimality: $\delta \equiv 0$; that's why we usually descend the square loss $\delta^2$
- Experience with high error $\Rightarrow$ more important to see
- Prioritized Experience Replay:
  - Sample instance $i$ with prob. $p_i \propto \delta_i^\omega$; e.g. $\omega = 0.6$
  - Update with Importance Sampling (IS) weight $(m \cdot p_i)^{-\beta}$; e.g. $\beta = 0.4$
- $\delta$ is computed during the updates; new experience is weighted $\max_i \delta_i^\omega$

[Schaul et al., 2016]

# Dueling Networks

- Advantage function: $A_\pi(s,a) = Q_\pi(s,a) - V_\pi(s)$
- $A_\pi(s,a)$ can be more consistent across states with similar effect of actions
  - Even if their value $V_\pi(s)$ is very different
- $V_\pi(s)$ is a scalar, which can be easier to learn




- Issue: $Q = (V + c) + (A - c)$ is underdetermined
  - Stabilize with $Q(s,a) = V(s) + \left(A(s,a) - \operatorname{mean}_{\bar a} A(s,\bar a)\right)$

[Wang et al., 2016]

# Multi-step Q Learning

- MC is high variance but unbiased: $Q(s_t, a_t) \to R_{\geq t} = \sum_{t' \geq t} \gamma^{t'-t} r_{t'}$

- TD is lower variance but biased: $Q(s_t, a_t) \to r_t + \gamma \max_{a_{t+1}} Q(s_{t+1}, a_{t+1})$
    - Because $\max_{a_{t+1}} Q(s_{t+1}, a_{t+1})$ isn't really the next-step value, while still learning

- Let's trade them off, $n$-step Q-Learning:

$$Q(s_t, a_t) \to r_t + \gamma r_{t+1} + \cdots + \gamma^{n-1} r_{t+n-1} + \gamma^n \max_{a_{t+n}} Q(s_{t+n}, a_{t+n})$$

# Rainbow DQN

- Rainbow DQN = DQN + a powerful combination of tricks
  - Double Q-Learning
  - Prioritized Experience Replay
  - Dueling Networks
  - Multi-step Q-Learning
  - Distributional RL
  - Noisy Nets

[Hessel et al., 2018]

# Recap

- RL algorithms can be implemented with function approximation
- There are (at least) 2 important policies
  - The learner policy — should be the best possible (e.g. greedy)
  - The experience policy — should explore (e.g. $\epsilon$-greedy)
- Replay buffer: store data for longer (off-policy), diversify
- Target network: reduce variance, stabilize the target
- In practice, add lots of tricks and heuristics to the theory